

Exercise 5B

1 a The gradient = -2

b The gradient = -1

c The gradient = 3

d The gradient = $\frac{1}{3}$

e The gradient = $-\frac{2}{3}$

f The gradient = $\frac{5}{4}$

g $2x - 4y + 5 = 0$

$$2x + 5 = 4y$$

$$4y = 2x + 5$$

$$y = \frac{2}{4}x + \frac{5}{4}$$

$$y = \frac{1}{2}x + \frac{5}{4}$$

The gradient = $\frac{1}{2}$

h $10x - 5y + 1 = 0$

$$10x + 1 = 5y$$

$$5y = 10x + 1$$

$$y = \frac{10}{5}x + \frac{1}{5}$$

$$y = 2x + \frac{1}{5}$$

The gradient = 2

i $-x + 2y - 4 = 0$

$$2y - 4 = x$$

$$2y = x + 4$$

$$y = \frac{1}{2}x + 2$$

The gradient = $\frac{1}{2}$

j $-3x + 6y + 7 = 0$

$$6y = 3x - 7$$

$$y = \frac{3}{6}x - \frac{7}{6}$$

$$y = \frac{1}{2}x - \frac{7}{6}$$

The gradient = $\frac{1}{2}$

k $4x + 2y - 9 = 0$

$$2y - 9 = -4x$$

$$2y = -4x + 9$$

$$y = -\frac{4}{2}x + \frac{9}{2}$$

$$y = -2x + \frac{9}{2}$$

The gradient = -2

l $9x + 6y + 2 = 0$

$$6y + 2 = -9x$$

$$6y = -9x - 2$$

$$y = -\frac{9}{6}x - \frac{2}{6}$$

$$y = -\frac{3}{2}x - \frac{1}{3}$$

The gradient = $-\frac{3}{2}$

2 a $c = 4$

b $c = -5$

c $c = -\frac{2}{3}$

d $y = -3x$

$$y = -3x + 0$$

$$c = 0$$

e $c = \frac{7}{5}$

f $y = 2 - 7x$

$$y = -7x + 2$$

$$c = 2$$

g $3x - 4y + 8 = 0$

$$3x + 8 = 4y$$

$$4y = 3x + 8$$

$$y = \frac{3}{4}x + \frac{8}{4}$$

$$y = \frac{3}{4}x + 2$$

$$c = 2$$

$$\begin{aligned}
 2 \text{ h } \quad 4x - 5y - 10 &= 0 \\
 4x - 10 &= 5y \\
 5y &= 4x - 10 \\
 y &= \frac{4}{5}x - \frac{10}{5} \\
 y &= \frac{4}{5}x - 2 \\
 c &= -2
 \end{aligned}$$

$$\begin{aligned}
 \text{i } \quad -2x + y - 9 &= 0 \\
 y - 9 &= 2x \\
 y &= 2x + 9 \\
 c &= 9
 \end{aligned}$$

$$\begin{aligned}
 \text{j } \quad 7x + 4y + 12 &= 0 \\
 4y + 12 &= -7x \\
 4y &= -7x - 12 \\
 y &= -\frac{7}{4}x - \frac{12}{4} \\
 y &= -\frac{7}{4}x - 3 \\
 c &= -3
 \end{aligned}$$

$$\begin{aligned}
 \text{k } \quad 7x - 2y + 3 &= 0 \\
 7x + 3 &= 2y \\
 2y &= 7x + 3 \\
 y &= \frac{7}{2}x + \frac{3}{2} \\
 c &= \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{l } \quad -5x + 4y + 2 &= 0 \\
 4y + 2 &= 5x \\
 4y &= 5x - 2 \\
 y &= \frac{5}{4}x - \frac{2}{4} \\
 y &= \frac{5}{4}x - \frac{1}{2} \\
 c &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ a } \quad y &= 4x + 3 \\
 0 &= 4x + 3 - y \\
 4x + 3 - y &= 0 \\
 4x - y + 3 &= 0
 \end{aligned}$$

$$\begin{aligned}
 3 \text{ b } \quad y &= 3x - 2 \\
 0 &= 3x - 2 - y \\
 3x - 2 - y &= 0 \\
 3x - y - 2 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{c } \quad y &= -6x + 7 \\
 6x + y &= 7 \\
 6x + y - 7 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{d } \quad y &= \frac{4}{5}x - 6 \\
 \text{Multiply each term by 5:} \\
 5y &= 4x - 30 \\
 0 &= 4x - 30 - 5y \\
 4x - 30 - 5y &= 0 \\
 4x - 5y - 30 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{e } \quad y &= \frac{5}{3}x + 2 \\
 \text{Multiply each term by 3:} \\
 3y &= 5x + 6 \\
 0 &= 5x + 6 - 3y \\
 5x + 6 - 3y &= 0 \\
 5x - 3y + 6 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{f } \quad y &= \frac{7}{3}x \\
 \text{Multiply each term by 3:} \\
 3y &= 7x \\
 0 &= 7x - 3y \\
 7x - 3y &= 0
 \end{aligned}$$

$$\begin{aligned}
 \text{g } \quad y &= 2x - \frac{4}{7} \\
 \text{Multiply each term by 7:} \\
 7y &= 14x - 4 \\
 0 &= 14x - 4 - 7y \\
 14x - 4 - 7y &= 0 \\
 14x - 7y - 4 &= 0
 \end{aligned}$$

3 h $y = -3x + \frac{2}{9}$

Multiply each term by 9:

$$9y = -27x + 2$$

$$27x + 9y = 2$$

$$27x + 9y - 2 = 0$$

i $y = -6x - \frac{2}{3}$

Multiply each term by 3:

$$3y = -18x - 2$$

$$18x + 3y = -2$$

$$18x + 3y + 2 = 0$$

j $y = -\frac{1}{3}x + \frac{1}{2}$

Multiply each term by 6 (6 is the LCM of 3 and 2):

$$6y = -2x + 3$$

$$2x + 6y = 3$$

$$2x + 6y - 3 = 0$$

k $y = \frac{2}{3}x + \frac{5}{6}$

Multiply each term by 6 (6 is the LCM of 3 and 6):

$$6y = 4x + 5$$

$$0 = 4x + 5 - 6y$$

$$4x + 5 - 6y = 0$$

$$4x - 6y + 5 = 0$$

l $y = \frac{3}{5}x + \frac{1}{2}$

Multiply each term by 10 (10 is the LCM of 5 and 2):

$$10y = 6x + 5$$

$$0 = 6x + 5 - 10y$$

$$6x + 5 - 10y = 0$$

$$6x - 10y + 5 = 0$$

4 $y = 6x - 18$

Substitute $y = 0$:

$$6x - 18 = 0$$

$$6x = 18$$

$$x = 3$$

The line meets the x -axis at $P(3, 0)$.

5 $3x + 2y = 0$

$$2y = -3x$$

$$y = -\frac{3}{2}x$$

The line meets the x -axis at $y = 0$.

Substituting $y = 0$ into $y = -\frac{3}{2}x$:

$$0 = -\frac{3}{2}x$$

$$x = 0$$

The line meets the x -axis at $R(0, 0)$.

6 $5x - 4y + 20 = 0$

Substitute $x = 0$:

$$5(0) - 4y + 20 = 0$$

$$-4y + 20 = 0$$

$$20 = 4y$$

$$4y = 20$$

$$y = 5$$

The line meets the y -axis at $A(0, 5)$.

Substitute $y = 0$:

$$5x - 4(0) + 20 = 0$$

$$5x + 20 = 0$$

$$5x = -20$$

$$x = -4$$

The line meets the x -axis at $B(-4, 0)$.

7 a The line passes through points $(0, 5)$ and $(6, 7)$.

$$\text{The gradient} = \frac{7-5}{6-0}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

b So $y = \frac{1}{3}x + c$

Use the point $(0, 5)$.

Substitute $x = 0$ and $y = 5$ into

$y = \frac{1}{3}x + c$ to find c .

$$5 = \frac{1}{3}(0) + c$$

$$c = 5$$

So $y = \frac{1}{3}x + 5$

$$3y = x + 15$$

$$x - 3y + 15 = 0$$

- 8 a** The line passes through $(5, 0)$ and $(0, 2)$.

$$\begin{aligned} \text{The gradient} &= \frac{2-0}{0-5} \\ &= -\frac{2}{5} \end{aligned}$$

b $y = -\frac{2}{5}x + c$

From the coordinates $(0, 2)$ the y -intercept is 2.

$$\text{So } y = -\frac{2}{5}x + 2$$

$$5y = -2x + 10$$

$$2x + 5y - 10 = 0$$

9 $ax + by + c = 0$

$$by = -ax - c$$

$$y = -\frac{a}{b}x - \frac{c}{b}$$

$$\text{The gradient} = -\frac{a}{b}$$

$$\text{The } y\text{-intercept} = -\frac{c}{b}$$

10 $ax - 2y + c = 0$

$$2y = ax + c$$

$$y = \frac{a}{2}x + \frac{c}{2}$$

$$\text{The gradient} = \frac{a}{2} = 3, \text{ so } a = 6.$$

$$\text{The } y\text{-intercept} = \frac{c}{2} = 5, \text{ so } c = 10.$$

$$a = 6, c = 10$$

11 Line l has equation $y = -2x + 6$.

For the second line:

$$5x - 8y - 15 = 0$$

$$8y = 5x - 15$$

$$y = \frac{5}{8}x - \frac{15}{8}$$

The lines intersect where:

$$y = -2x + 6 \text{ and } y = \frac{5}{8}x - \frac{15}{8} \text{ cross}$$

$$\text{So } -2x + 6 = \frac{5}{8}x - \frac{15}{8}$$

Multiply through by 8.

$$-16x + 48 = 5x - 15$$

$$21x = 63$$

$$x = 3$$

Substituting $x = 3$ into $y = -2x + 6$:

$$y = -2(3) + 6$$

$$y = 0$$

The lines intersect at $P(3, 0)$.

12 a $l_1: y = 3x - 7$

The point of intersection is $P(-3, b)$.

Substituting $x = -3$ and $y = b$ into l_1 :

$$b = 3(-3) - 7$$

$$b = -16$$

b $l_2: ax + 4y - 17 = 0$

The point $P(-3, -16)$ is on the line.

Substituting $x = -3$ and $y = -16$ into l_2 :

$$a(-3) + 4(-16) - 17 = 0$$

$$-3a - 81 = 0$$

$$a = -27$$

Challenge

The line passes through $(0, a)$ and $(b, 0)$.

$$\begin{aligned} \text{The gradient} &= \frac{0-a}{b-0} \\ &= -\frac{a}{b} \end{aligned}$$

The y -intercept is at $x = 0$.

This is the point $(0, a)$.

$$y = -\frac{a}{b}x + a$$

$$by = -ax + ab$$

$$ax + by - ab = 0$$